Screening Task
for
P(11-35) - Project Research Assistant

Please attempt all questions. Remember that your submission will be graded on the quality of your work and validity of the results.

Task

Please see the subsequent pages and attempt all the 5 questions.

Submission procedure for the Task:

1. Put your code along with other supporting files (if any) in a folder. You may put a README file inside the folder, if you want, to give us more information about your submission. Rename that folder as “first_name-job_code”, without quotes. For example, **P(11-35)-satish**
2. Compress the folder in ZIP format. Avoid any other compression format.
3. Mail the zip file to **info@fossee.in.**

Make sure to put the mail subject-line as “**job-code-YourName** without quotes. For example, “**P(11-35)-satish**” No extension in the deadline will be considered for submission of screening task

Consider a set of defferential equations:

The reaction $A \rightarrow B$ takes place in a continuous stirred-tank reactor (CSTR). The reaction is exothermic in nature.

$$\frac{dC_A}{dt} = \frac{F(C_{A0} - C_A)}{V} - 2kC_A^2$$

$$\frac{dT}{dt} = \frac{F(T_0 - T)}{V} - \frac{2(\Delta H)kC_A^2}{\rho C_p} - \frac{U_0 A(T - T_j)}{V\rho C_p}$$

$C_A \rightarrow$ concentration of A in reactor

$T \rightarrow$ temperature of mixture in reactor

$F \rightarrow$ feed flow rate

C_{A0}, k, T_0, V, ΔH, ρ, C_p, U_0, A, T_j, V_p are constants.

Assume $\frac{\Delta H}{\rho C_p}$ as 'a', $\frac{U_0 A}{V_p C_p}$ as 'b'

$C_{A0} = 1$, $V = 100$, $k = 4.11 \times 10^{13}$, $T_0 = 275$, $\rho = 1$, $-\Delta H = 596619$, $C_p = 4200$, $U_0 A = 20000 \times 60$, $T_j = 250$

Modified set of equations :

$$\frac{dC_A}{dt} = \frac{F(C_{A0} - C_A)}{V} - 2kC_A^2$$

$$\frac{dT}{dt} = \frac{F(T_0 - T)}{V} - 2kaC_A^2 - b(T - T_j)$$

where, state vector $X = [C_A \quad T]^T$

Manipulated input vector $U = F$

Measured variable $Y = T$

Above set of equations can be written as:

$$\frac{dX}{dt} = f(X, U)$$

with $Y = g(X) \cancel{= [0 \quad 1] X}$

__Question 1:__ Develop Linear perturbation model at steady-state conditions $\bar{X}$, $\bar{U}$

$$\bar{X} = [0.0192 \quad 384.05]^T$$

$$\bar{U} = 120$$

Use following formulae for it:

$$\frac{dX}{dt} = AX + BU \quad , \cancel{y = Cx}$$

$$Y = CX$$

$A = \left[\frac{\partial f}{\partial x}\right]_{n \times n}$ at $\bar{x}, \bar{U}$ $n =$ number of states (2 in our case)

↑ Jacobian

$B = \left[\frac{\partial f}{\partial U}\right]_{n \times m}$ at $\bar{x}, \bar{U}$ $m =$ number of inputs (1 in our case)

↑ Jacobian

$C = \left[\frac{\partial g}{\partial x}\right]_{r \times n}$ at $\bar{x}, \bar{U}$ $r =$ number of outputs (1 in our case)

Question 2: Convert continuous-time linear model to discrete-time linear model.

$$\left.\begin{aligned} \frac{dX}{dt} &= AX + BU \\ Y &= CX \end{aligned}\right\} \rightarrow \text{Continuous-time}$$

$$x(k+1) = \phi\, x(k) + \Gamma\, u(k) \quad \rightarrow \text{Discrete-time}$$

Use following formulae:

$$\phi = e^{AT_s}$$

$$\Gamma = \int_0^{T_s} e^{A\tau} B \, d\tau$$

where $T_s \rightarrow$ sampling time, $T_s = 0.1$ min

Question 3: Consider an 'ARX' model:

$$y(k) + a_1 y(k-1) = b_1 u(k-1) + b_2 u(k-2) + e(k)$$

where $e(k)$ is a white-noise sequence, $u(k)$ and $y(k)$ are time-domain series (input-output data)

$$A\, y(k) = B\, u(k) + e(k)$$

where $A = (1 + a_1 q^{-1})$, $B = (b_1 q^{-1} + b_2 q^{-2})$

Write this polynomial form of 'ARX' in 'Observable' and 'controllable' state-space form.

Question 4: How will you find the values of parameters (a_1, b_1, b_2) if u and y vectors are provided to you.

$u(1), u(2) \ldots \ldots u(10)$
$y(1), y(2) \ldots \ldots y(10)$ } values for these are provided to you

Note:
(Write the method/approach you will follow).

Question 5: Considering you are provided with input-output data (y–u vectors), write a code sequence of MATLAB/SCILAB/OCTAVE to get 'e' vector in terms of parameters(a_1, b_1, b_2).